Mathematics Framework Solution Sets
Grade 5

A note about these solutions.

These solutions are intended for teachers, not students. The solutions are fairly detailed and some include additional comments that serve to further explain the content and purpose of each problem. It is important to note that these solutions are not meant to be representative of student solutions.

It is the nature of many mathematics problems that they can be solved in different ways. In general, the solutions given here represent simply one way of solving the problems. At times, a second solution path is offered.

It is our hope that these solution sets will help teachers to better see the essential skills and concepts that are important to student success in Grade 5 mathematics.

Number Sense 1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

Problem: Change to decimals: $\frac{17}{1,000}$, $\frac{3}{20}$, 6%, $35\frac{1}{2}$.

Solution: We have

$$\begin{aligned}
\frac{17}{1,000} &= 0.017 \\
\frac{3}{20} &= \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15 \\
6\% &= 6 \times \frac{1}{100} = \frac{6}{100} = 0.06 \\
35\frac{1}{2} &= 35.5
\end{aligned}$$

Problem: Change to fractions: 0.03, 1.111, 8%, 21

Solution: We have

$$\begin{aligned}
0.03 &= \frac{3}{100} \\
1.111 &= 1 + 0.111 = 1 + \frac{111}{1,000} = \frac{1,000}{1,000} + \frac{111}{1,000} = \frac{1,111}{1,000} \\
8\% &= 8 \times \frac{1}{100} = \frac{8}{100} = \frac{2 \times 4}{25 \times 4} = \frac{2}{25} \\
21 &= \frac{21}{1}
\end{aligned}$$

Number Sense 1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

Problem: Change to percents: 0.07, 0.165, $\frac{17}{20}$, $\frac{1}{8}$.

Solution: We have

$$\begin{aligned}
0.07 &= \frac{7}{100} = 7 \times \frac{1}{100} = 7\% \\
0.165 &= \frac{165}{1000} = \frac{165}{10} \times \frac{1}{100} = 16.5 \times \frac{1}{100} = 16.5\% \\
\frac{17}{20} &= \frac{17 \times 5}{20 \times 5} = \frac{85}{100} = 85 \times \frac{1}{100} = 85\% \\
\frac{1}{8} &= 0.125 = \frac{125}{1000} = \frac{125}{10} \times \frac{1}{100} = 12.5 \times \frac{1}{100} = 12.5\%
\end{aligned}$$

Further Explanation: A percent represents some part of a whole, expressed with the whole being 100. In other words, 22% can be thought of as expressing the ratio 22 out of 100. The term percent refers to "per one hundred," and the symbol "%" can be considered to represent "$\times \frac{1}{100}$".
If a and b are numbers, then the percent b that a represents can be found by setting up the proportion

$$\frac{a}{b} = \frac{n}{100},$$

and solving for n. The critical thinking here is that of proportional reasoning: whether expressed as a/b or $n/100$, the ratio represented is the same.

Number Sense 1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

Problem: These are the next three problems:

(i) 6 is what % of 25?

(ii) What is 15% of 44?

(iii) 30 is 20% of what?

Solution: (i) Since 6 is $\frac{6}{25}$ of 25, we have

$$\frac{6}{25} = \frac{6 \times 4}{25 \times 4} = \frac{24}{100} = 24\%.$$

Therefore, 6 is 24% of 25. Another way to solve this is to set up the proportion

$$\frac{6}{25} = \frac{n}{100},$$

where n represents the percentage. In this case, we have

$$600 = 25n \quad \Rightarrow \quad n = 24.$$

(ii) 15% of 44 is the number that is a fraction of $\frac{15}{100}$ of 44. In other words, since

$$\frac{15}{100} \times 44 = \frac{660}{100} = 6.6,$$

we have that 6.6 is 15% of 44. As before, a common way of solving this is to set up the proportion

$$\frac{a}{44} = \frac{15}{100},$$

where a is the unknown part of 44 we seek. Solving this yields $a = 6.6$.
(iii) We are looking for a number b such that 20% of it is 30. In other words, $0.2 \times b = 30$. Dividing, we can find b:

$$b = \frac{30}{0.2} = \frac{30}{2/10} = \frac{300}{2} = 150.$$

Therefore, 30 is 20% of 150. Again, we can also solve this by setting up the proportion

$$\frac{30}{b} = \frac{20}{100},$$

where b is the unknown number of which 30 is a part, and solving for b. Solving this yields $b = 150$.

Number Sense 1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

Problem: Betty paid \$23.60 for an item that was reduced by 20%.

1. What was the original price?
2. If the original price was reduced by 25%, what is the sale price?

Solution: 1. Let P represent the original price. Then Betty paid only $(100-20)\% = 80\%$ of P. As in previous problems, 80% of P can be represented by

$$\frac{80}{100} \times P = 0.8P.$$

Since she paid \$23.60 for the item, we can set up the equation

$$0.8P = 23.60 \quad \Rightarrow \quad P = 23.60 \div 0.8 = 29.50.$$

Therefore, the original price of the item was \$29.50.

2. If the item was discounted 25%, then it costs 75% of \$29.50. Let S represent the sale price. Then

$$S = 0.75P = 0.75 \times 29.50 = 22.125 \approx 22.13,$$

so the sale price is \$22.13.

Number Sense 1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$**).**

Problem: Write as a product of primes using exponents (use factor trees or other methods): 18, 48, 100

Solution: We have

$$
\begin{aligned}
18 &= 2 \times 9 = 2 \times 3 \times 3 = 2 \times 3^2 \\
48 &= 4 \times 12 = 2 \times 2 \times 4 \times 3 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3 \\
100 &= 4 \times 25 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2
\end{aligned}
$$

Further Explanation: The Fundamental Theorem of Arithmetic states that each natural number $n > 1$ can be uniquely expressed as a product of prime numbers. Finding prime factorizations is a crucial skill in working with both whole numbers and rational numbers. Factoring polynomials, which comes later in the mathematics curriculum, is an abstraction of this process.

Number Sense 1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication.
Number Sense 1.4 Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$**).**

Problem: What is the largest square of a whole number that divides 48? What is the largest cube of a whole number that divides 48?

Solution: From the previous problem, we have the prime factorization of 48: $48 = 2^4 \times 3$. Any number that is a factor of 48 has 2 and 3 as prime factors, with exponents no larger than their exponents in the factorization of 48. We know that any number to an even exponent is a perfect square. Since 2 appears in this factorization with an exponent of 4, we can take $16 = 2^4$ as the largest perfect square that divides 48. Similarly, any number to a power that is a multiple of 3 is a cube. Again, looking at the factorization of 48, we see that $8 = 2^3$ is the largest cube that divides 48.

Number Sense 1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

Problem: Arrange in order from smallest to largest: $\frac{9}{4}$, 25%, 0.3, $2\frac{1}{2}$, 0.295

Solution: If we convert each number into a decimal, we can easily order them. We have

$$\begin{aligned}\frac{9}{4} &= \frac{8}{4} + \frac{1}{4} = 2 + 0.25 = 2.25 \\ 25\% &= \frac{25}{100} = 0.25 \\ 0.3 &= 0.3 \\ 2\frac{1}{2} &= 2 + \frac{1}{2} = 2 + 0.5 = 2.5 \\ 0.295 &= 0.295\end{aligned}$$

Since

$$0.25 < 0.295 < 0.3 < 2.25 < 2.5,$$

we see that the order is

$$25\% < 0.295 < 0.3 < \frac{9}{4} < 2\frac{1}{2}.$$

Number Sense 2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Problem: Find the average of $6.81, 7, 5.2$ and round the answer to the nearest hundredth.

Solution: To find the average of three numbers, we add them and divide by 3:

$$\text{average} = \frac{6.81 + 7 + 5.2}{3} = \frac{19.01}{3} = 6.33\overline{6} \approx 6.34.$$

Further Explanation: In general, to find the average of N numbers, we add them and divide by N. This average is also called the *arithmetic mean.*

Problem: Evaluate $0.25(3 - 0.75)$.

Solution: We solve in two ways. First, we apply order of operations and subtract in the parenthesis:

$$\begin{aligned} 0.25(3 - 0.75) &= 0.25(2.25) \\ &= .5625 \end{aligned}$$

Alternatively, we apply the distributive property:

$$\begin{aligned} 0.25(3 - 0.75) &= (0.25)(3) - (0.25)(0.75) \\ &= 0.75 - 0.1875 \\ &= 0.5625 \end{aligned}$$

Moreover, one can evaluate the expression by first converting to fractions and then applying similar reasoning:

$$\begin{aligned} 0.25(3 - 0.75) &= \left(\frac{1}{4}\right)\left(3 - \frac{3}{4}\right) \\ &= \left(\frac{1}{4}\right)\left(\frac{12}{4} - \frac{3}{4}\right) \\ &= \left(\frac{1}{4}\right)\left(\frac{9}{4}\right) \\ &= \frac{9}{16}. \end{aligned}$$

Number Sense 2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Problem: Do the following problems mentally:

$$\left(\frac{9,185}{2,117} \times \frac{12}{13}\right) \div \frac{9,185}{2,117} =?$$
$$\left(\frac{9,185}{13} \times \frac{12}{2,117}\right) \div \frac{9,185}{2,117} =?$$

Solution: The first problem is simply an instance of

$$A \times B \div A = \frac{AB}{A} = B,$$

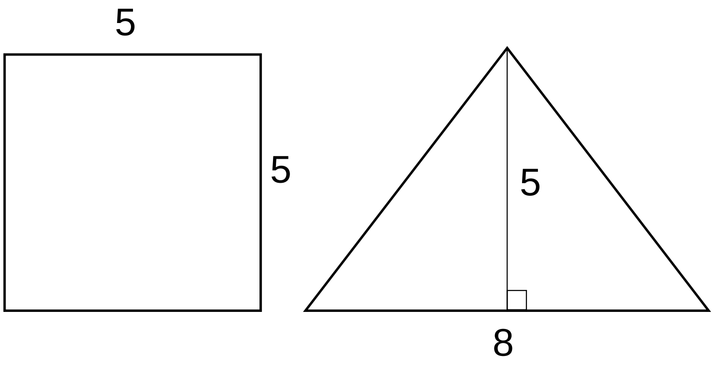
so that the answer to the first problem is $\frac{12}{13}$.
For the second problem, notice that for $b, d \neq 0$,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a}{d} \times \frac{c}{b},$$

so that we can interchange the denominators 13 and 2,117 to obtain the first problem. The answer is also $\frac{12}{13}$.

Measurement and Geometry 1.0 Students understand and compute the volumes and areas of simple objects.

Problem: Find the areas (dimensions are in cm):

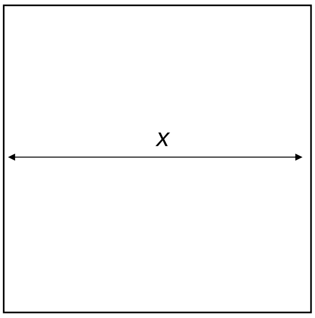


Solution: The area of a square can be found by squaring the side length. Thus, the area of the first shape is $5^2\,\text{cm}^2 = 25\,\text{cm}^2$.
The area of a triangle is given by the formula $\frac{1}{2}bh$, where b is a base length, and h the height of the triangle with respect to this base. In the picture, the side of length 8 cm is a suitable base with the height being 5 cm. The area is therefore

$$\frac{1}{2}(8)(5)\,\text{cm}^2 = 4 \cdot 5\,\text{cm}^2 = 20\,\text{cm}^2.$$

Measurement and Geometry 1.1 Derive and use the formulas for the area of a triangle and of a parallelogram by comparing with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).

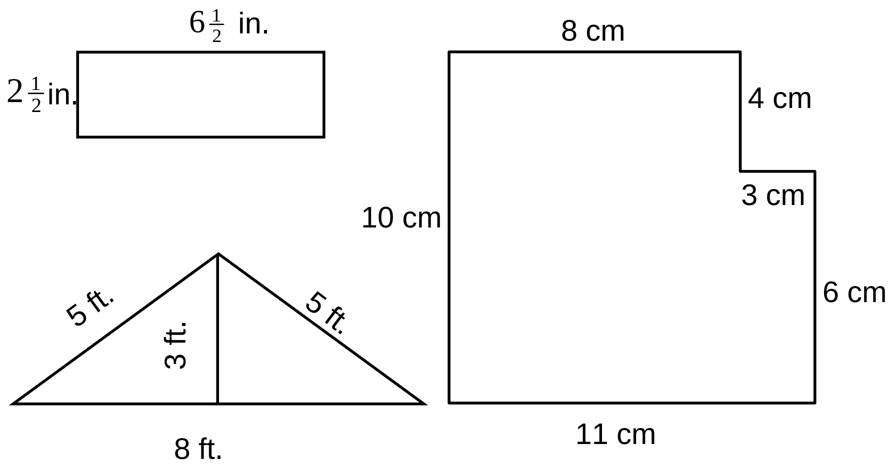
Problem: How many segments x will fit on the perimeter of the square?



Solution: The width of the square is indicated as x, so the perimeter of the square is $4x$ units. This means that 4 segments of length x units will fit around the square.

Measurement and Geometry 1.1 Derive and use the formulas for the area of a triangle and of a parallelogram by comparing with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).

Problem: Find the area of each figure.



Solution: The area of the rectangle shown is

$$\begin{aligned} 6\frac{1}{2} \times 2\frac{1}{2}\,\text{in}^2 &= \frac{13}{2} \times \frac{5}{2}\,\text{in}^2 \\ &= \frac{65}{4}\,\text{in}^2 \\ &= 8\frac{1}{4}\,\text{in}^2. \end{aligned}$$

The area of the isosceles triangle is

$$\frac{1}{2}(8)(3)\,\text{ft}^2 = 12\,\text{ft}^2.$$

The area of the irregular shape can be found in several ways. One way is to see it as a 10 cm by 8 cm rectangle connected to a 6 cm by 3 cm rectangle. The total area in that case is

$$(10 \times 8 + 6 \times 3)\,\text{cm}^2 = 98\,\text{cm}^2.$$

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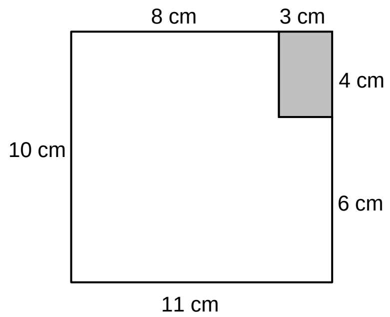
Another way to do the problem is to find the area of a large 10 cm by 11 cm rectangle and subtract the area of a "removed" 3 cm by 4 cm rectangle:



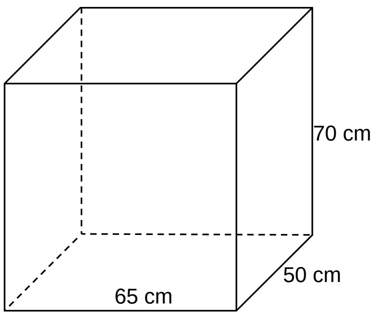
The result is

$$(110 - 12)\,\text{cm}^2 = 98\,\text{cm}^2.$$

Measurement and Geometry 1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [in^3], cubic yard [yd^3]) to compute the volume of rectangular solids.
Measurement and Geometry 1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

Problem: Determine the volume of a rectangular solid with length 65 cm, height 70 cm, and width 50 cm. For the same rectangular solid, determine its surface area. (Make sure that your answer is expressed in the correct units.)

Solution: A picture is shown below:



The volume of this rectangular prism is

$$65 \times 70 \times 50\,\text{cm}^3 = 227,500\,\text{cm}^3.$$

The surface area is found by finding the areas of each of the 6 faces of the shape and adding these up. There are two 65 cm by 70 cm faces, two 70 cm by 50 cm faces, and two 65 cm by 50 cm faces. This gives us the following expression for the surface area:

$$[2(65 \times 70) + 2(70 \times 50) + 2(65 \times 50)]\,\text{cm}^2.$$

Simplifying, we have

$$[2(4,550) + 2(3,500) + 2(3,250)]\,\text{cm}^2 = 2(11,300)\,\text{cm}^2 = 22,600\,\text{cm}^2.$$

Measurement and Geometry 1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

Problem: Identify the relevant dimension as length, area, or volume:

1. The perimeter of a triangle
2. The capacity of a barrel
3. The capacity of a box
4. The amount of sod needed to cover a football field
5. The number of bricks needed to pave a path
6. The height of a tree

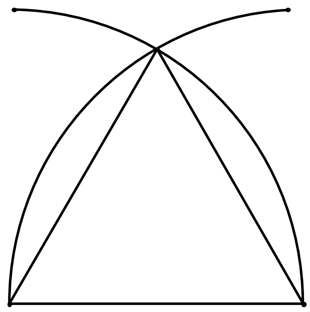
Solution: We have:

1. Perimeter measures the length of the edges of a polygon or the length around a shape. The dimension is length.
2. Capacity is a measurement of volume.
3. This is volume. (In fact, with a box, it may be easier to show why the measurement is cubic units.)
4. This is a measurement of area; it is describing the amount of "flat" space enclosed by a linear border.
5. If seen as similar to the sod question above, we are asking about area. However, it seems that one would find the area needed to be covered, the area that each brick covers, and *then* the number of bricks. (The number of bricks itself is not a measurement of area.)
6. The height of a tree is a measurement of length.

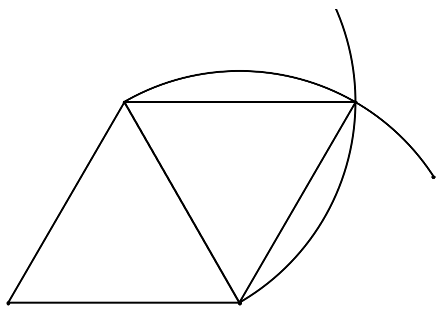
Measurement and Geometry 2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

Problem: Explain how to make the following basic constructions with a straight edge and compass; e.g., an equilateral triangle, a regular hexagon, a line passing through a given point and perpendicular to a given line.

Solution: To construct an equilateral triangle of a given side length, we start with the point of a compass at one endpoint of the segment and draw a circular arc of radius the length of the segment. Then we do the same at the other endpoint. A point where these two arcs meet will serve as the third vertex of an equilateral triangle.



To construct a regular hexagon, one could repeat the above construction using a newly created side of the triangle and simply continue around until a regular hexagon was obtained.



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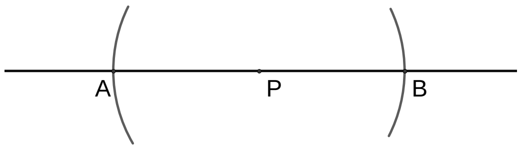
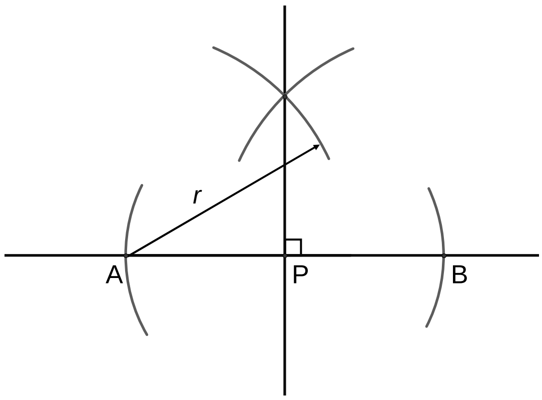
To construct a line through a given point perpendicular to a given line, we consider two cases. First suppose the point P is on the line. Then we will make P the midpoint of a segment by constructing a circle with P as the center. This circle intersects the line in two points A and B and such that P is the midpoint of AB.



Next, we choose a radius r greater than the length of AP and construct two small arcs of this radius, one centered at A and one centered at B. The line through the intersection point of these arcs and P is the perpendicular bisector of the segment AB and the line we seek.



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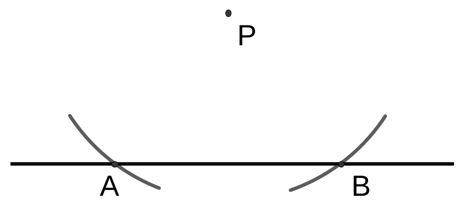
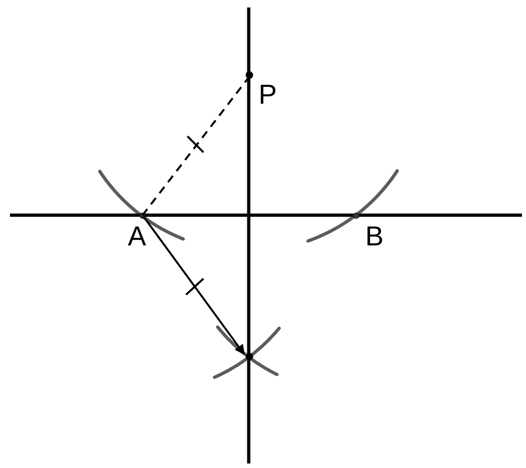
If P does not lie on the line, then we take a circular arc centered at P of sufficient radius to intersect the line in points A and B.



Then, keeping the radius the same, we construct two small arcs, one centered at A and the other centered at B on the other side of the line with respect to P. The line through the intersection of these arcs and P will work.

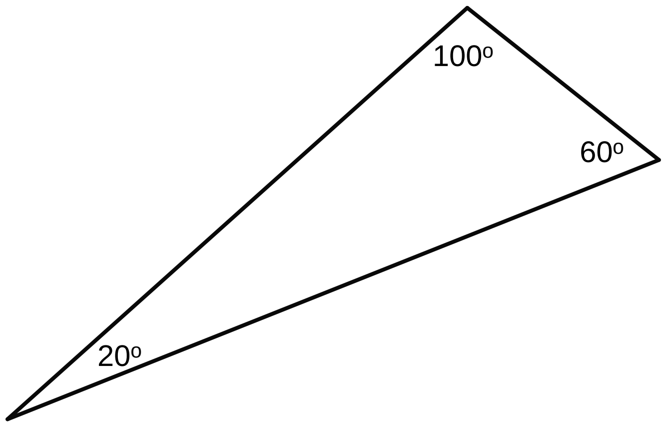


Measurement and Geometry 2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.

Problem: Find the third angle of a triangle if you know that one angle is 60° and the second angle is 20°.

Solution: Since the sum of the measures of the angles of a triangle is 180°, we find that the missing angle measure is

$$180^\circ - (60^\circ + 20^\circ) = 100^\circ.$$



Statistics Data Analysis and Probability 1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

Problem: Draw a circle graph to display the following data: A certain municipal district spends 6 million dollars per year–$2,507,000 on education, $1,493,000 for public safety, $471,000 for libraries, $536,000 for road maintenance, and $993,000 for miscellaneous expenses. (This problem also applies to Number Sense Standards 1.1 and 1.2.)

Solution: If we take the data and find the approximate percentages each expense represents out of the total $6,000,000 spent annually, then we can easily create a circle graph to represent the data.
We have

Expense	$$$	$\approx$ Pct.
education	$ 2,507,000	41%
public safety	$ 1,493,000	25%
library	$ 471,000	8%
roads	$ 536,000	9%
miscellaneous	$ 993,000	17%
total	$ 6,000,000	100%

The circle graph is given below.